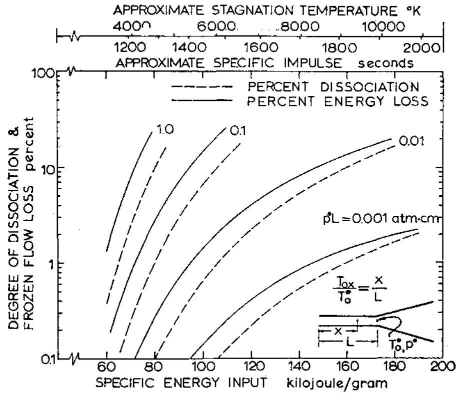

Fig. 1 Estimated dissociation and frozen flow loss for a nozzle with constant area heat addition

not substantially different from the conclusions in Table 2.

The nozzle and heat input distribution used in this calculation are not substantially different from some actual nozzles in current use. Indeed, nonequilibrium heating actually may be taking place in some arc-jet engine nozzles, which usually operate with pL of the order of unity.[4]

Concluding Remarks

Although the preliminary calculations presented in the foregoing indicate that nonequilibrium heating is feasible, more detailed investigations of the kinetics and aerodynamics are necessary to evaluate quantitatively its effectiveness in reducing frozen flow loss. In particular, the influence of electron impact dissociation[7] at high electron concentrations and temperatures should be investigated. It is hoped that this note will stimulate interest among propulsion engineers in these endeavors.

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On Two Alternative Motivations of Reference-State Expressions for Turbulent Flows with Mass Transfers

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For turbulent flows without mass transfers, reference-temperature (or reference-enthalpy) expressions obtained using two alternative motivations are found to be equivalent to good approximation. For example, Eckert,[1] guided apparently by the expression evolved empirically for laminar flows, finds that

$$T^* = 0.50\,T_w + 0.50\,T_\infty + 0.22\,r_0{}^*u_\infty{}^2/2c_p{}^* \tag{1}$$

correlates satisfactorily data for turbulent flows over flat plates; Burggraf,[2] equating the reference enthalpy and the enthalpy at the outer edge of the laminar sublayer, obtains an expression that may be written (for constant specific heat)

$$T^* = \left(1 - \frac{Pr^*}{s_{h0}{}^*}\frac{u_{s0}}{u_\infty}\right)T_w + \frac{Pr^*}{s_{h0}{}^*}\frac{u_{s0}}{u_\infty}T_\infty + \frac{Pr^*}{s_{h0}{}^*}\frac{u_{s0}}{u_\infty}\left(1 - \frac{s_{h0}{}^*}{r_0{}^*}\frac{u_{s0}}{u_\infty}\right)\frac{r_0{}^*u_\infty{}^2}{2c_p{}^*} \tag{2}$$

where T is temperature, u velocity parallel to surface, c_p constant-pressure specific heat, Pr Prandtl number, r recovery factor, s_h Reynolds-analogy factor for heat transfer, subscripts w, s, ∞, and 0 refer, respectively, to wall, outer edge of laminar sublayer, outer edge of boundary layer, and zero-blowing conditions, and the asterisk indicates that the property is to be evaluated at the reference state. For

$$\frac{Pr^*}{s_{h0}{}^*}\frac{u_{s0}}{u_\infty} = \frac{s_{h0}{}^*}{r_0{}^*}\frac{u_{s0}}{u_\infty} = \frac{1}{2}$$

(a typical value), Eqs. (1) and (2) are equivalent to good approximation. The purpose of the present note is to compare extensions of Eqs. (1) and (2) to the case of turbulent flows with mass transfers.

Expressions for the reference state (reference temperature and reference composition) for laminar boundary layer flow with mass transfer have been developed by Knuth.[3] The development of the reference-temperature expression was guided by an analysis of Couette flow in which the variation of the specific heat with composition was taken into account. The purposes of the present note are satisfied, however, if one uses the simplified expression obtained by applying the methods of Ref. 3 to a model in which the specific heat is constant with a value fixed by the reference state. One obtains

$$T^* = 0.50\,T_w + 0.50\,T_\infty + 0.17\,r_0{}^*u_\infty{}^2/2c_p{}^* + 0.08(c_p{}^c/c_p{}^*)\,Pr^*B_f{}^*(T_w - T_\infty) \tag{3}$$

where

$$B_f{}^* \equiv [(\rho v)_w/\rho^*u_\infty](2/C_{f0}{}^*)$$

and where ρ is density, v is velocity normal to surface, C_f is friction coefficient, and superscript c refers to coolant. The expression for reference composition involves, in general, logarithmic terms; the linearized expression

$$c^{c*} = 0.50\,c_w{}^c + 0.50\,c_\infty{}^c \tag{4}$$

may be used, however, for low blowing rates or small molecular-weight differences. In a study of the limited available data, it was concluded[4] that, for Mach numbers up to 3 and for injection of nitrogen into air, Eq. (3) correlates satisfactorily the data for turbulent boundary layer flows with mass transfers.

In the alternative motivation of reference-state expressions for turbulent flows with mass additions, one would equate the reference temperature and composition to the temperature and composition at the outer edge of the laminar sublayer. For a model in which the specific heat is constant with value fixed by the reference state, an expression for the temperature at the outer edge of the laminar sublayer is given by Eq. (15) of Ref. 4. Solving for T_s,

$$T_s = T_w - \frac{r_s{}^*u_s{}^2}{2c_p{}^*} + \frac{k^*(dT/dy)_w}{(\rho v)_w c_p{}^c}\left[\left(\frac{\tau_s}{\tau_w}\right)^{(c_p{}^c/c_p{}^*)Pr^*} - 1\right]$$

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where k is thermal conductivity, τ shearing stress, and y coordinate normal to stationary surface. If one substitutes for $k^*(dT/dy)_w$ and τ_s/τ_w from

$$k^*(dT/dy)_w \equiv C_h^*\rho^*u_\infty c_p^*(T_\infty + r^*u_\infty^2/2c_p^* - T_w)$$

and

$$\tau_s/\tau_w = 1 + (\rho v)_w\, u_s/\tau_w$$

then

$$T_s = T_w - \frac{r_s^*u_s^2}{2c_p^*} + C_h^* \frac{\rho^*u_\infty c_p^*}{(\rho v)_w c_p^c} \times \left[\left(1 + \frac{(\rho v)_w u_s}{\tau_w}\right)^{(c_p^c/c_p^*)Pr^*} - 1\right] \times \left(T_\infty + \frac{r^*u_\infty^2}{2c_p^*} - T_w\right)$$

Use of this equation is facilitated if the several factors and coefficients appearing in the right-hand side are expressed in terms of zero-blowing factors (or coefficients) and the blowing rate. As indicated in Ref. 3, additional convenience is realized with apparently negligible sacrifice in utility if one expands in series and retains only terms up to and including first-order terms in blowing rates. Then

$$C_h^* \frac{\rho^*u_\infty c_p^*}{(\rho v)_w c_p^c}\left[\left(1 + \frac{(\rho v)_w u_s}{\tau_w}\right)^{(c_p^c/c_p^*)Pr^*} - 1\right] \approx \frac{C_h^*Pr^*}{C_f^*/2}\frac{u_s}{u_\infty}\left[1 + \frac{1}{2}\left(\frac{c_p^c}{c_p^*}Pr^* - 1\right)B_f^*\frac{u_s}{u_\infty}\right]$$

whereas the exact expression for r_s^* given by Ref. 4 becomes

$$\frac{r_s^*}{Pr^*} \approx 1 + \frac{1}{3}\left(\frac{c_p^c}{c_p^*}Pr^* - 1\right)B_f^*\frac{u_s}{u_\infty}$$

Expressions for the Reynolds-analogy factor $C_f^*/2C_h^*$ and the recovery factor r^* are more complicated. Simplifications are realized, however, and the purposes of the present note are served, if the turbulent Prandtl number is equated to the laminar Prandtl number. Then Eqs. (20) and (21) of Ref. 4 may be written as

$$\frac{C_h^*s_{h0}^*}{C_f^*/2} \approx 1 - \frac{1}{2}\left(\frac{c_p^c}{c_p^*}Pr^* - 1\right)B_f^*$$

and

$$\frac{r^*}{r_0^*} \approx 1 + \frac{1}{3}\left(\frac{c_p^c}{c_p^*}Pr^* - 1\right)B_f^*$$

Combining Eqs. (3) and (5) of Ref. 4 and linearizing,

$$\frac{u_s}{u_{s0}} \approx \left(\frac{C_f}{C_{f0}}\right)^{1/4} \approx 1 - \frac{1}{4}B_f^*\frac{u_{s0}}{u_\infty}$$

Finally, substituting these five linearized expressions into the equation for T_s and setting T_s equal to the reference temperature,

$$T^* = \left(1 - \frac{Pr^*}{s_{h0}^*}\frac{u_{s0}}{u_\infty}\right)T_w + \frac{Pr^*}{s_{h0}^*}\frac{u_{s0}}{u_\infty}T_\infty + \frac{Pr^*}{s_{h0}^*}\frac{u_{s0}}{u_\infty} \times \left(1 - \frac{s_{h0}^*}{r_0^*}\frac{u_{s0}}{u_\infty}\right)\frac{r_0^*u_\infty^2}{2c_p^*} + \frac{1}{2}\frac{Pr^*}{s_{h0}^*}\frac{u_{s0}}{u_\infty}\left(1 - \frac{u_{s0}}{u_\infty}\right) \times \frac{c_p^c}{c_p^*}Pr^*B_f^*(T_w - T_\infty) - \frac{1}{4}\frac{Pr^*}{s_{h0}^*}\frac{u_{s0}}{u_\infty} \times \left(2 - 3\frac{u_{s0}}{u_\infty}\right)B_f^*(T_w - T_\infty) - \frac{1}{6}\frac{Pr^*}{s_{h0}^*}\frac{u_{s0}}{u_\infty} \times \left[1 - 3\frac{u_{s0}}{u_\infty} + 2\frac{s_{h0}^*}{r_0^*}\left(\frac{u_{s0}}{u_\infty}\right)^2\right]\left(\frac{c_p^c}{c_p^*}Pr^* - 1\right) \times B_f^*\frac{r_0^*u_\infty^2}{2c_p^*} - \frac{1}{4}\frac{Pr^*}{s_{h0}^*}\left(\frac{u_{s0}}{u_\infty}\right)^2\left(1 - 2\frac{s_{h0}^*}{r_0^*}\frac{u_{s0}}{u_\infty}\right) \times B_f^*\frac{r_0^*u_\infty^2}{2c_p^*} \quad (5)$$

In a similar treatment of the reference composition, beginning with Eq. (14) of Ref. 4, one obtains

$$c^{c*} = \left(1 - \frac{Sc^*}{s_{m0}^*}\frac{u_{s0}}{u_\infty}\right)c_w^c + \frac{Sc^*}{s_{m0}^*}\frac{u_{s0}}{u_\infty}c_\infty^c \quad (6)$$

where Sc is Schmidt number and s_m is Reynolds-analogy factor for mass transfer. For

$$\frac{Pr^*}{s_{h0}^*}\frac{u_{s0}}{u_\infty} = \frac{Sc^*}{s_{m0}^*}\frac{u_{s0}}{u_\infty} = \frac{s_{h0}^*}{r_0^*}\frac{u_{s0}}{u_\infty} = \frac{1}{2}$$

and

$$u_{s0}/u_\infty = \tfrac{1}{2}$$

(typical values), Eq. (5) becomes

$$T^* = 0.50\,T_w + 0.50\,T_\infty + 0.25\,r_0^*u_\infty^2/2c_p^* + 0.11\,(c_p^c/c_p^*)Pr^*B_f^*(T_w - T_\infty) - 0.04\,B_f^*(T_w - T_\infty) + 0.01[(c_p^c/c_p^*)Pr^* - 1]B_f^* \times (r_0^*u_\infty^2/2c_p^*)$$

whereas Eq. (6) becomes identically Eq. (4). Hence, also for the case with mass transfer, the difference between the reference-state expressions obtained using the two alternative motivations appears to be small.

As a consequence of the strong dependence of Schmidt number upon composition, the most sensitive comparison of the results of these two alternative motivations might be realized in an effort to correlate mass-transfer rates for cases in which the coolant molecular weight differs greatly from the mainstream-gas molecular weight. Knuth[3] found, for laminar flows with high rates of injection of hydrogen or iodine into air, that the expression

$$1 - c^{c*} = \frac{M^a}{M^a - M^c}\frac{\ln M_\infty/M_w}{\ln c_\infty^a M_\infty/c_w^a M_w} \quad (7)$$

where M is molecular weight, is required. Results of efforts using this equation might be compared with results of efforts using

$$c^{c*} = \left(1 - \frac{Sc^*}{s_{m0}^*}\frac{u_{s0}}{u_\infty}\right)c_w^c + \frac{Sc^*}{s_{m0}^*}\frac{u_{s0}}{u_\infty}c_\infty^c + \frac{1}{2}\frac{Sc^*}{s_{m0}^*}\frac{u_{s0}}{u_\infty}\left(1 - \frac{u_{s0}}{u_\infty}\right)Sc^*B_f^*(c_w^c - c_\infty^c) - \frac{1}{4}\frac{Sc^*}{s_{m0}^*}\frac{u_{s0}}{u_\infty}\left(2 - 3\frac{u_{s0}}{u_\infty}\right)B_f^*(c_w^c - c_\infty^c) \quad (8)$$

obtained by equating the coolant concentration at the outer edge of the laminar sublayer to the reference coolant concentration and retaining second-order terms in blowing rates. Unfortunately, the data (including composition at the wall) required for this comparison do not appear to be available at present. Perhaps this note will encourage some reader, having the necessary experimental facilities, to make the required measurements.

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Method for Measuring Damping about the Input Axis of a Single-Degree-of-Freedom Floated Gyro

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Nomenclature

D_i = damping about input axis
D_0 = damping about output axis
H = angular momentum
i = current into torquer
I_i = inertia about input axis
I_0 = inertia about output axis
K_0 = spring constant about axis
K_p = pickoff constant
K_T = torquer scale factor
T_A = torque applied = iK_T
T_D = restraint torques
$\dot{\phi}$ = float rate about input axis
ψ = angular position of float about output axis
$\dot{\psi}$ = float rate about output axis
$\ddot{\psi}$ = float acceleration about output axis

WITH new and more sophisticated uses for high-precision gyroscopes, more critical examination of their behavior is required. For example, gyroscopic devices commonly are classified as either single-degree-of-freedom or two-degree-of-freedom units. In actuality, under fine analysis all gyroscopic devices exhibit two-degree-of-freedom performance. This is true for the single-degree-of-freedom gyro because of the clearance requirements in the pivot and jewel assembly that supports the structure containing the gyro rotor (Fig. 1). This clearance allows the float to rotate about its input axis, causing it to behave, under certain conditions, as a two-degree-of-freedom gyro.

This behavior exists only when the gyro senses a change in an applied rotation, and it ceases when the clearance is taken up. Although the effect is small since the angular clearance in a precision gyro is usually less than 1 min of arc, it can prove troublesome. It is of particular importance in inertial platform stabilization loops. Under the proper combination of small input angle excursions, it can result in improper stabilization due to gyro nutation and limit cycle effects.

Another area of concern is in the application of single-degree-of-freedom gyros to body-mounted guidance systems. Here, the gyro dynamics about the input axis produce a lag in the rate-measuring process, causing the introduction of cross-coupling errors. Thus, in order to evaluate and to simulate properly the behavior of a single-degree-of-freedom gyro in a sophisticated system, the dynamics of the gyro about its input axis must be known.

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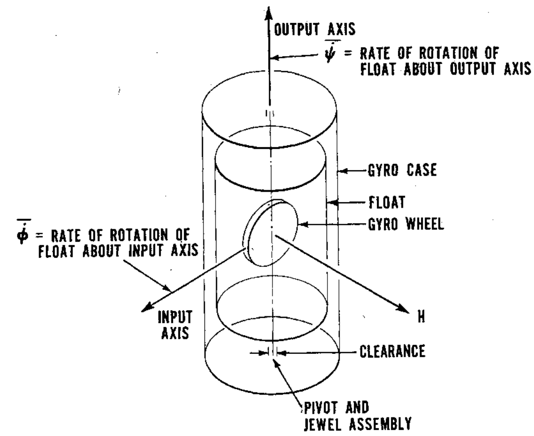

Fig. 1 Single-degree-of-freedom floated rate integrating gyro (actually, the float has movement about the input axis as well as about the output axis)

Two factors, inertia and damping, make up the input axis dynamics. The former can be calculated; the latter has been estimated by means of analytical computations but, to the author's knowledge, never has been measured experimentally. This article describes in detail an experimental procedure for measuring damping about the input axis of a single-degree-of-freedom floated gyro.

Figure 2 shows the equipment necessary to measure input axis damping. None of this equipment is itself novel; it is standard test equipment available wherever gyro evaluation is taking place. What is new is the method of using the equipment and the calculations shown here. The only information required is the output axis damping and the angular momentum of the gyro.

In this method, an accurately known current is put into the gyro torquer which sets up a known force on the float about the output axis, causing the latter to rotate. This can be described mathematically by the equation

$$iK_T = I_0\ddot{\psi} + D_0\dot{\psi} + K_0\psi \tag{1}$$

Since the spring constant in rate-integrating gyros is kept extremely small, Eq. (1) can be rewritten as

$$iK_T = T_A = I_0\ddot{\psi} + D_0\dot{\psi} \tag{2}$$

In Laplace notation

$$T_A(s) = (I_0S + D_0)\dot{\psi}(s) \tag{3}$$

Solving for the output axis rate,

$$\dot{\psi}(s) = \frac{T_A(s)}{I_0S + D_0} = \frac{T_A(s)/D_0}{(I_0/D_0)S + 1} \tag{4}$$

It should be noted that Eq. (4) holds true only when no pivot clearance exists. Actually, pivot clearances do exist,

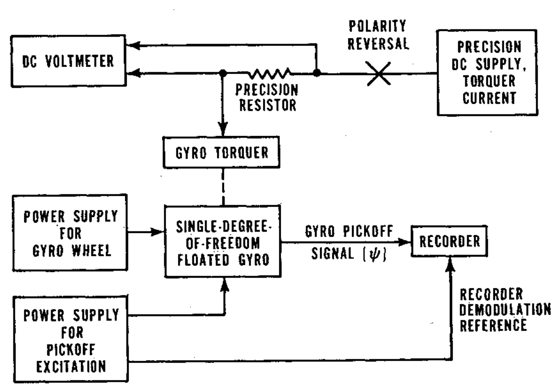

Fig. 2 Equipment necessary to measure damping about the input axis of a single-degree-of-freedom floated gyro